AN INTERPOLATION/EXTRAPOLATION APPROACH TO X-RAY IMAGING OF SOLAR FLARES

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Abstract. We describe an interpolation/extrapolation procedure that reconstructs X-ray maps of solar flares using as input data sparse samples of the Fourier transform of the radiation flux, named visibilities. The algorithm is based on two steps: in the first step the performance of an interpolation routine is optimized by representing the visibilities according to favorable coordinates in the frequency plane. In the second step two extrapolation schemes are introduced, respectively based on the projection and the thresholding of the Landweber iterative method. The procedure is validated against realistic synthetic visibilities and applied to experimental measurements provided by the NASA satellite Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI).

1. Introduction. The Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) [14] has been launched by NASA on 2002 with the aim of providing X-ray observations of eruptive events in the solar atmosphere with unprecedented spectral and spatial resolution. Specifically, RHESSI data are utilized to infer information on the physics of solar flares, with particular focus on the acceleration mechanisms of electrons in the solar plasma, at the basis of X-ray emission. Compared to previous solar hard X-ray satellites like, for example the Solar and Heliospheric Observatory (SOHO), RHESSI can extend its observations in the energy range from 3 keV to 1 MeV (rather than from 15 to 100 keV), detect variation in the energy spectra within 1 keV (rather than 5 keV) and reach an angular resolution of 2 arcseconds.
(rather than 5 arcseconds). For all these reasons, although ten years after its launch, RHESSI can be still considered the most up-to-date currently operating solar X-ray mission and will represent the inescapable scientific background for upcoming devices like the American Gamma-Ray Imager/Polarimetry for Solar Flares (GRIPS) or the European Spectrometer/Telescope Imaging X-rays (STIX) in Solar Orbiter.

RHESSI imaging works by image modulation rather than by focusing, in a way recalling how interferometric radio telescopes image the sky. The rotation of the grid pairs of each one of its nine Rotating Modulation Collimators (RMCs) provides a temporal modulation of the incoming flux and the pattern of such temporal modulation provides not an image but rather a specific set of spatial Fourier components of the source, called visibilities [11]. These visibilities, which are complex numbers with corresponding complex error bars, are fully calibrated, containing no instrumental dependence other than instrumentally defined spatial frequencies, are not biased by background and, thanks to symmetry properties of the imaging system, offer a level of redundant information that can be exploited as indication of systematic errors.

Image reconstruction from visibilities has a well-established history in radioastronomy, where visibilities are generated by interferometric telescopes. However, in that framework the coverage of the frequency plane is typically much denser than in the case of RMCs for X-ray imaging [19]. Therefore several methods for the processing of X-ray visibilities have been recently developed [1, 18, 15]. In particular, the method named 'uv_smooth' [15], realizes a two-step strategy: first an interpolation of the visibilities in the frequency space provides a visibility surface which, in the second step, is inverted by means of an iterative algorithm with a positivity constraint. An implementation of the method is part of the Solar SoftWare tree including software packages for data analysis in solar astronomy and is currently used to reconstruct images of solar flares from RHESSI data [12, 17]. In the present paper we provide a general mathematical framework to 'uv_smooth' and develop it with the aim of enhancing the performances of both the interpolation and the extrapolation steps. Specifically, in the first step, the interpolation process is optimized by representing the measured visibilities according to specific coordinates that make optimal the sampling of the frequency plane. In the second step, we compare the performances of two different kinds of constraints on the solution in order to obtain super-resolution effects. More precisely:

- in the first approach we utilize a level set routine [7] on the map resulting from the inverse Fourier transform of the interpolated visibilities and realize an extrapolating constraint by means of a projected Landweber method, where the projection is onto the set of compactly supported positive functions [16];
- in the second approach, we search for a sparse reconstruction by applying a soft thresholding technique which is again implemented by means of a modification of the Landweber iterative scheme [9].

A systematic validation of this interpolation/extrapolation approach by means of synthetic and real visibilities allows us to show its advantages, which are numerous and rather clear: it is an FFT-based procedure and therefore it is notably rapid; it does not need any data pre-processing such as background subtraction; it is very robust with respect to different source configurations and, thanks to its flexibility and speed, can be used for bootstrap analysis. The implementation of this new algorithm in the Solar SoftWare (SSW) tree of the RHESSI mission is under construction.
The plan of the paper is as follows. Section 2 briefly describes the visibility concept; sections 3 and 4 will discuss the interpolation and extrapolation procedures at the basis of our method; section 5 will perform a validation of the method by means of realistic simulations and two real observations. Our conclusions will be offered in section 6.

2. **Visibilities.** We describe the visibility concept within the framework of the RHESSI instrument. However, it is straightforward to extend such description to other X-ray or gamma-ray telescopes like STIX or GRIPS. The RHESSI imaging system is characterized by a rather simple geometry: nine high-sensitivity Germanium detectors are utilized to detect the photons coming from the Sun; each detector is associated with a pair of co-axial collimators; each collimator is made of a planar array of equally-spaced X-ray-opaque slats separated by transparent slits; the nine pairs of grids are characterized by nine different widths for the slits; finally, the overall system rotates with a rotation period of 4 seconds and, furthermore, the satellite rotates around the Earth at an orbit of around 600 km. RHESSI raw data are light curves, i.e. photon-induced counts recorded while the collimators rotate. A technical procedure made of a data stacking step followed by a fitting step transforms these raw data into a set of observable numbers, named *visibilities*, that represent the purest way with which RHESSI may provide its measurements [11]. RHESSI visibilities are complex numbers representing measurements of single Fourier components of the source distribution measured at specific spatial frequencies, energy ranges and time ranges. If \((u, v)\) represents a point in the spatial frequency plane and \(\epsilon\) is a specific photon energy (or, more typically, an energy channel spanning a specific energy bin), the corresponding measured visibility \(V(u, v; \epsilon)\) is related to the photon flux \(I(x, y; \epsilon)\) emitted from the point \((x, y)\) of the imaging plane by

\[
V(u, v; \epsilon) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(x, y; \epsilon) e^{2\pi i (ux + vy)} \, dx \, dy. \tag{1}
\]

The values of the sampled frequency pair \((u, v)\) are determined by RHESSI hardware. Specifically, the collimators-detector system characterized by the narrowest width of the slits (named Collimator 1) provides the best resolution limit (but the worst signal-to-noise ratio) while the collimators-detector system characterized by the largest width of the slits (named Collimator 9) provides the worst resolution limit (but the best signal-to-noise ratio). Coherently, as shown in Figure 1(a), the visibilities associated to Collimator 1 sample the frequency \((u, v)\) plane on a circle with maximum radius \(R_1 = 0.221 \text{ arcsec}^{-1}\) while the visibilities associated to Collimator 9 sample the \((u, v)\) plane on a circle with minimum radius \(R_9 = 0.0027 \text{ arcsec}^{-1}\). Because the grid pitches of RHESSI’s collimators are arranged in a geometric progression with factor \(\sqrt{3}\), the radii \(R_i, i = 1, \ldots, 9\) of these circles in the spatial frequency plane are also distributed in a geometric progression with the same factor. Therefore, in summary, RHESSI is a Fourier imaging system that, overall, measures the Fourier transform of the photon flux at points over nine circles in the frequency plane and the RHESSI imaging problem is the inverse Fourier transform problem with limited data of reconstructing meaningful images of the photon flux from these sparsely sampled Fourier components.

3. **Interpolation.** Armed with RHESSI measured visibilities, one can reconstruct images of the source by simply inverse Fourier transforming them from the frequency to the physical imaging plane. However, such an approach is of course limited by
the sparse sampling of the frequency plane realized by RHESSI collimators. This frequency information gap between the nine visibility circles can be filled up rather naturally by means of an interpolation procedure. Interpolation, indeed, generates a smooth continuum of visibilities within the disk \( \Omega_1 \) of radius \( R_1 \) corresponding to the area of the \((u,v)\) plane encompassed by Collimator 1. Once this visibility surface has been created, an inverse Fourier transform technique can be applied to reconstruct the physical image.

In order to make this approach effective, two practical issues must be accounted for. First, the sampling points must be uniformly distributed over the \((u,v)\) plane in order to make FFT-based inversion routines immediately applicable. Second, RHESSI observations sample the \((u,v)\) plane in a very sparse way and therefore a representation of the spatial frequencies must be searched for, that optimizes the accuracy of the interpolation. The scheme we use to account for such issues and to realize an effective interpolation of the visibility values is described in Figure 1 and is based on the following steps:

1. the \((u,v)\) plane is sampled according to a uniform grid of \( N \times N \) points, containing \( \Omega_1 \) (white points in Figure 1(b)), i.e.
   \[
   u_i = u_0 + i \Delta u \quad i = 0, \ldots, N - 1
   \]
   \[
   v_j = v_0 + j \Delta v \quad j = 0, \ldots, N - 1
   \]
   In our applications we will use \( N = 450, \quad u_0 = v_0 = -0.255 \text{ arcsec}^{-1}; \quad \Delta u = \Delta v = 0.00025 \text{ arcsec}^{-1}; \)

2. the sampling points (i.e., the points in the \((u,v)\) plane chosen in the previous step) are represented in a new plane according to the new coordinates
   \[
   \theta_{ij} = \arctan \frac{v_i}{u_j}; \quad \log \rho_{ij} = \log \sqrt{u_i^2 + v_j^2}
   \]
   with \( i, j = 0, \ldots, N - 1 \) (white points in Figure 1(c)). Accordingly, the experimental knots (i.e., the positions in the \((u,v)\) plane of the experimental visibilities) are represented with the \((\theta, \log \rho)\) coordinates as well (red crosses in Figure 1(c)). With this transformation the sampling points are no longer uniformly distributed, but the experimental knots on which the interpolation is performed are now equispaced with respect to the \(\log \rho\) coordinate;

3. just the sampling points \((\theta, \log \rho)\) such that \( \log R_9 \leq \log \rho \leq \log R_1 \) are used for the interpolation (these are the white points in Figure 1(d));

4. the interpolation routine is applied in the \((\theta, \log \rho)\) plane. The resulting interpolated surface is sampled in correspondence with the white points and 'back-represented' in the \((u,v)\) space. An example of the output of this procedure is in Figure 1(e), representing the amplitude of the interpolated visibilities corresponding to an off-center two-dimensional Gaussian function (Figure 1(f) is a zoomed version of this visibility surface).

In step 4 of the previous scheme we applied the two-dimensional thin-plate spline [5], implemented in SSW, which is a generalization of cubic spline.

As shown in Figure 1(e) and (f), the interpolation scheme just sketched does not provide a smooth visibility surface. The reason for this drawback is two-fold. First, the discontinuity on the surface (see Figure 1(e)) is due to the fact that the result of interpolation is not necessarily the same at \( \theta = 0 \) and \( \theta = 2\pi \). Hence, when the output of the interpolation is projected back to the \((u,v)\) plane this reflects into a non-smooth refolding of the visibility surface. Second, the hole at \((u,v) = (0,0)\) (see
Figure 1. Interpolation of RHESSI visibilities corresponding to an off-center Gaussian source: (a) sampling of the \((u,v)\) plane performed by RHESSI collimators; (b) uniform square sampling grid (white points) containing the circles in the \((u,v)\) plane spanned by the collimators; (c) representation of the sampling points (white points) and of the experimental knots (red crosses) in the \((\theta, \log \rho)\) plane; (d) sampling points utilized for the interpolation; (e) visibility surface provided by the interpolation; (f) zoom on the surface highlighting the discontinuity at \(\theta = 0\) and the hole at low frequency.
Figure 1(f), which is a zoom of Figure 1(e), is due to the fact that information on the visibilities at the origin of the $(u, v)$ plane should be provided by interpolation values at $\log \rho = -\infty$ in the $(\theta, \log \rho)$ and, obviously, no value is available there (the points with the smallest $\log \rho$ values are those on the line $\log \rho = \log R_9$). These drawbacks can be overcome by means of two simple expedients which are illustrated in Figure 2(a). In order to eliminate the discontinuity on the visibility surface, the interpolation is performed between $\theta = -2\pi$ and $\theta = 4\pi$, where the new set of knots to interpolate corresponds to straightforward replicas of the original experimental knots. In this way, the newly interpolated values at $\theta = 0$ and $\theta = 2\pi$ are constrained to coincide by the smoothing properties imposed by the spline interpolation. On the other hand, the central hole is filled by adding a virtual collimator with radius $R_{10}$, equal to the one of the smallest circle we can construct inside the image (i.e., half the sampling distance in the $(u, v)$ plane) and providing it with constant visibility values corresponding to the DC component of the total flux (i.e., the maximum of the Fourier transform of the flux). Of course then we sample the interpolation surface at points such that $\log R_{10} \leq \log \rho \leq \log R_1$. The result of the new interpolation is zoomed in Figure 2(b), clearly showing that both the refolding discontinuity and the lack of information at the origin of the $(u, v)$ plane are restored.

As shown in [15], for many flaring configurations (for examples, in the case of single compact sources or extended loops) the interpolation step is rather robust with respect to the kind of interpolation routine applied. However, for very specific configurations of the flaring source, the oscillations of the experimental visibilities may be rather wild and in these cases the choice of the interpolation procedure may become rather critical. This situation typically occurs when the emitting region is not connected and each connected component is far from the other more than 20 arcsec. A way to overcome this drawback will be briefly discussed in the Conclusions.
4. Extrapolation. The interpolation procedure described in the previous section provides a set of visibilities uniformly sampled in the disk $\Omega_1$ of radius $R_1 = 0.221$ arcsec$^{-1}$ in the $(u, v)$ plane. Therefore the available (measured + interpolated) visibility data $\hat{g}(u, v)$ are related to the spatial distribution $f(x, y)$ of the count flux by means of

$$\hat{g}(u, v) = \begin{cases} \hat{f}(u, v) + \hat{e}(u, v) & (u, v) \in \Omega_1 \\ 0 & (u, v) \in \mathbb{R}^2 \setminus \Omega_1, \end{cases}$$

where the error term $\hat{e}(u, v)$ resembles the measurement uncertainty on the visibilities. It follows that

$$g(x, y) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} e^{-i2\pi(xu+vy)} \hat{g}(u, v) dudv$$

is a noisy, bandlimited approximation of $f(x, y)$. This implies two important drawbacks of model (5): first, the Shannon sampling theorem shows that the best spatial resolution achievable by the hardware is given by the Nyquist distance $\Lambda = \pi/R_1$, i.e. no detail smaller than $\Lambda$ can be seen by RHESSI detectors; second, any imaging method based on a naive inverse Fourier transform algorithm will provide reconstructions characterized by ringing effects [13].

The problem of extrapolating information on $f(x, y)$ out of the band $\Omega_1$ cannot be addressed without additional information on $f(x, y)$. An effective way to do this is to represent the out-of-band extrapolation problem within the framework of operator theory in Hilbert spaces. To this aim, let us introduce the linear operator $A : L^2(D) \to L^2(\mathbb{R}^2)$ such that

$$(Af)(x, y) = \int_{D} H_{\Omega_1}(x-x', y-y') f(x', y') dxdy'.$$

where $D$ is the compact support of $f(x, y)$, $H_{\Omega_1}(x, y)$ is the band limited function

$$H_{\Omega_1}(x, y) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} \chi_{\Omega_1}(u, v) e^{-2\pi i (ux+vy)} dudv$$

and $\chi_{\Omega_1}(u, v)$ is the characteristic function of the band $\Omega_1$. Since $A$ is compact, the linear inverse problem of estimating the source function $f^{(0)}$ by solving

$$g = Af$$

when

$$g = Af^{(0)} + e,$$

is ill-posed in the sense of Hadamard. To infer some information on $\hat{f}$ out of the band $\Omega_1$, one should constrain the space of feasible solutions. This can be achieved by constraining the approximate solution to belong to the subspace of compactly supported functions, by imposing a positivity constraint or by requiring the approximate solution to be sparse. In the following we describe two modifications of the Landweber iterative scheme [4] that provide out-of-band extrapolation and sparsity-enhancement effects.

4.1. Projected iterations. The solution of problem (10) can be achieved iteratively, by means of the projected Landweber method [16]

$$f_0 = 0$$

$$f_{n+1} = P_D[f_n + \tau A^*(g - Af_n)],$$
where $A^*$ is the adjoint operator of $A$, $\tau$ is fixed between 0 and 2 and $P_D$ is the projector

$$ (P_D f)(x, y) = \begin{cases} 0 & x \notin D \\ f(x, y) & \text{otherwise} \end{cases}. $$

(13)

The algorithm strongly converges in the case of noise-free data, while for noisy data the semi-convergence property holds in most practical cases, so that there is a unique optimal iteration number $n_{opt}$ for which the $L^2$ distance between $f_{n_{opt}}$ and the theoretical $f^{(0)}$ reaches a minimum value. Accelerations of the algorithm can be realized by both using a different initialization (for example, some estimate of the source obtained by means of some coarser method) or by preconditioning the original inverse problem as showed in [16]. Finally, out-of-band extrapolation is assured by the projection onto $D$, since constraining the solution to belong to a compact domain in $\mathbb{R}^2$ results in an analytic Fourier transform $\hat{f}$.

The implementation of the projected Landweber method by utilizing the Fourier transform of all the functions involved in the problem leads to the following version of the Gerchberg-Papoulis [10] method:

$$ f_0 = 0 $$

(14)

$$ \hat{f}_{n+1} = \tau \hat{g} + [1 - \tau \chi_{\Omega_1}] \hat{f}_n $$

(15)

$$ f_{n+1} = P_D f_{n+1}. $$

(16)

In the specific case of RHESSI data analysis (but, more generally, of astronomical imaging), the following two issues are also considered.

First, the pixel content in X-ray images is always non-negative and this information can be easily encoded in the reconstruction procedure by replacing $P_D$ in (13) with the projection operator $P_D^+$ such that

$$ (P_D^+ f)(x, y) = \begin{cases} 0 & (x, y) \notin D \text{ or } f(x, y) < 0 \\ f(x, y) & \text{otherwise} \end{cases}. $$

(17)

Second, an estimate of the support $D$ can be obtained by means of the following two steps:

1. the inverse Fourier transform of the visibility surface is realized by applying an FFT-based routine. This leads to a blurred reconstruction of the source function;

2. an edge detection procedure based on level sets [7] is applied to this coarse reconstruction in order to determine a coarse profile of the source. The support $D$ is then given by the binary mask which is 1 inside the profile and 0 outside.

4.2. **Thresholded iterations.** The crucial role of sparsity for obtaining out-of-band extrapolation has been already pointed out by Donoho in the case of diffraction imaging [8]. Following Daubechies, Defrise and De Mol [9] here we apply a straightforward sparsity promoting modification of the Landweber scheme to the reconstruction of X-ray images from RHESSI visibilities. If $t$ is a threshold value identified on a coarse reconstruction of the image, the thresholded iterative scheme is

$$ \hat{f}_{n+1} = \tau \hat{g} + [1 - \tau \chi_{\Omega_1}] \hat{f}_n $$

(18)

$$ f_{n+1} = P^+(f_{n+1} - t), $$

(19)

where

$$ (P^+) f(x, y) = \begin{cases} 0 & f(x, y) < 0 \\ f(x, y) & \text{otherwise} \end{cases}. $$

(20)
The value of $t$ can be determined as a fraction of the peak intensity in the inverse Fourier transform of the interpolated visibility surface.

4.3. **Implementation.** A stopping rule for both the projected and the thresholded iterative schemes is a heuristic implementation of the Morozov’s discrepancy principle [20], where the optimal number of iterations is given by the solution of the equation

$$
\| [\chi_{\Omega_1}, \hat{f}]_{\text{exp}} - [\hat{g}]_{\text{exp}} \|_2^2 = \Delta^2
$$

where $[\cdot]_{\text{exp}}$ denotes the fact that in the two matrices $\chi_{\Omega_1}, \hat{f}$ and $\hat{g}$ just the entries corresponding to the experimental frequencies sampled by RHESSI are considered (the other entries are set to zero), $\| \cdot \|$ is the corresponding Frobenius norm and $\Delta$ is the Frobenius norm of the error matrix.

The computational procedures described in the previous and present sections can be summarized in an image reconstruction scheme which transforms sparse and noisy samples of the Fourier transform of the X-ray source into a super-resolved ringing-free image of the flaring event. Specifically, here is our overall algorithm for X-ray imaging from RHESSI visibilities:

1. **Input data:** visibilities measured by RHESSI (one set of complex numbers representing the observations and one set of complex numbers representing the observation errors).

2. **Pre-processing:**
   - creation of a squared grid of uniformly distributed frequencies $(u_i, v_j)$, $i, j = 1, \ldots, N^2$, containing the disk $\Omega_1$, where $\Omega_1$ is delimited by the frequency circle sampled by Collimator 1;
   - $(\theta, \log \rho)$ representation of the frequencies in the grid and of the experimental frequencies sampled by RHESSI;
   - replica of the frequencies sampled by RHESSI and of the corresponding visibilities (originally observed in the $\theta$-interval $[0, 2\pi]$) in the intervals $[-2\pi, 0)$ and $(2\pi, 4\pi]$;
   - creation of a virtual collimator at $\log \rho = \log R_{10}$, provided with constant visibility values corresponding to the DC component, where $R_{10}$ is equal to half the sampling distance in the $(u, v)$ plane.

3. **Interpolation:** creation of the visibility surface by applying a thin-plate spline interpolation routine against the (original and replicated) visibilities in the $(\theta, \log \rho)$ plane plus the DC component at $\log \rho = \log R_{10}$ and ‘back-representation’ in the original $(u, v)$ space.

4. **Extrapolation:** super-resolution effects can be obtained by either:
   - application of an inverse Fourier transform routine to the visibility surface;
   - application of a level set algorithm to the resulting map and creation of a binary mask which is one inside the contour highlighted by the level set method and zero outside;
   - application of the projected Landweber method to the visibility surface, with $P^*_D$ as projection operator, whereby the support $D$ is provided by the mask computed at the previous step and the stopping rule is provided by the discrepancy principle;
   - application of the thresholded Landweber method to the visibility surface where the threshold is given by a fraction of the peak intensity in the inverse Fourier transform of the visibility surface.
For both iterative schemes the stopping rule is provided by the discrepancy principle.

5. Numerical validation. The validation of our algorithm is performed against synthetic visibilities computed according to a very realistic procedure. Specifically, in Figure 3 we describe two synthetic topographies similar to two real events observed by RHESSI: the February 20 2002 flare in the energy range from 22 keV to 26 keV and the July 23 2002 event in the energy range from 36 keV to 41 keV. These synthetic maps, characterized by intensity distributions comparable to the real ones, are given as input to an SSW routine computing a set of visibilities that realistically mimic the ones observed by RHESSI. In particular, in both cases this routine provides 64 visibilities for each collimator. Our algorithm is applied to these two sets of visibilities and the results of the imaging procedure are described in Figure 4 for the case inspired by the February 20 2002 event and in Figure 5 for the case inspired by the July 23 2002 event. For each figure, the four panels contain the real and imaginary parts of the interpolated visibility surface, the result of the reconstruction procedure when the projected iterations are applied and the result of the reconstruction procedure when the thresholded iterations are applied.

In order to quantitatively assess the reliability of the reconstruction procedure, in Table 1 and Table 2 we computed some geometric and physical parameters pertinent to the two configurations. For both cases we computed the overall root mean square error. Then we define the centroid \((x_C, y_C)\) of a map (or of a portion of an image) of intensity distribution \(I(x, y)\) as

\[
x_C = \frac{\int \int xI(x, y)dxdy}{\int \int I(x, y)dxdy},
\]

\[
y_C = \frac{\int \int yI(x, y)dxdy}{\int \int I(x, y)dxdy}.
\]

In the case of the map inspired by the February 20 2002 event, we constructed two Regions of Interest (ROIs) around the two sources and for each ROI we provided the distance between the theoretical and reconstructed centroid position and the relative error on the peak intensity. In the case of the map inspired by the July 23 2002 event, the ROIs we considered are four, coherently to the topography, and, again, for each ROI we computed the absolute errors on the centroid position and the relative error on the peak intensity. The two tables contain as well the same error analysis in the case of reconstructions of the same visibility sets, when the method applied is the maximum entropy algorithm implemented in SSW [6]. The two extrapolation schemes provide a similar accuracy, although the thresholded algorithm performs better than the projected one in the reconstruction of 9 parameters over 14. For both configurations the interpolation/extrapolation procedure provides a notably smaller root mean square error than maximum entropy, and this is due in particular to the inaccurate performance of maximum entropy in reconstructing the peak intensity of the sources. On the other hand, maximum entropy may be more precise in the reconstruction of the source positions. We also point out that the interpolation/extrapolation procedure is systematically more effective from a computational viewpoint: the reconstructions are obtained after a few seconds with a standard low cost CPU, while with the same hardware, it takes at least one minute to maximum entropy to converge.

As examples of the behavior of the method in the analysis of experimental visibilities, we show in Figure 6 the reconstructions corresponding to RHESSI visibilities
Table 1. Reconstruction errors for the simulated configuration inspired by the February 20 2002 event. The methods applied are our interpolation/extrapolation procedure (with projected and thresholded iterations) and a maximum entropy method: ‘rms’ denotes the root mean square error over the entire image; ‘pos N’ and ‘pos S’ are the distances between the original and reconstructed upper (North) and bottom (South) source centroids; ‘peak N’ and ‘peak S’ are the relative errors on the peak intensity for the same sources.

<table>
<thead>
<tr>
<th>Method</th>
<th>rms</th>
<th>pos N</th>
<th>pos S</th>
<th>peak N</th>
<th>peak S</th>
</tr>
</thead>
<tbody>
<tr>
<td>projected</td>
<td>0.26</td>
<td>0.20 arcsec</td>
<td>0.51 arcsec</td>
<td>0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>thresholded</td>
<td>0.22</td>
<td>0.19 arcsec</td>
<td>0.48 arcsec</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>mem</td>
<td>0.74</td>
<td>0.38 arcsec</td>
<td>0.30 arcsec</td>
<td>0.72</td>
<td>0.70</td>
</tr>
</tbody>
</table>

recorded during the 2004 May 21 flare (in the energy range from 10 to 15 keV) and during the 2005 August 23 event (in the energy range between 14 and 16 keV). In both cases the reconstructions provided by our method, both when projected iterations and thresholded iterations are applied, are realistic and can be easily motivated by means of classical models in solar flare physics.

![Theoretical maps](image)

**Figure 3.** Theoretical maps utilized for the validation of the reconstruction methods: (a) simulated configuration inspired by the February 20 2002 event; (b) simulated configuration inspired by the July 23 2002 event.

6. **Conclusions.** We have described a new interpolation/extrapolation procedure for image reconstruction from measurements provided by the NASA satellite RHESSI.
Figure 4. Application of the interpolation/extrapolation algorithm to synthetic visibilities corresponding to a flaring configuration inspired by the February 20 2002 real event: (a) real part of the interpolated visibility surface; (b) imaginary part of the interpolated visibility surface; (c) reconstruction provided by the interpolation/extrapolation method when the constrained Landweber scheme is applied; (d) reconstruction provided by the interpolation/extrapolation method when the thresholded Landweber scheme is applied.

Due to the hardware design of this instrument for solar X-ray observations, these data are Fourier components of the radiation flux, named visibilities, sampled over nine circles with nine different radius in the frequency plane (each circle corresponds to one of the nine RHESSI collimators). Therefore, after providing an efficient representation of these data using appropriate coordinates, we have interpolated the visibilities to obtain a smooth visibility surface which has been extrapolated outside
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Figure 5. Application of the interpolation/extrapolation algorithm to synthetic visibilities corresponding to a flaring configuration inspired by the July 23 2002 real event: (a) real part of the interpolated visibility surface; (b) imaginary part of the interpolated visibility surface; (c) reconstruction provided by the interpolation/extrapolation method when the constrained Landweber scheme is applied; (d) reconstruction provided by the interpolation/extrapolation method when the thresholded Landweber scheme is applied.

the frequency region covered by Collimator 1 (the collimator with the best spatial resolution) by means of a projected and a thresholded iterative scheme. The accuracy and effectiveness of this approach have been tested against both synthetic and real visibilities. In the case of synthetic data, a comparison with both the original theoretical configuration and with a maximum entropy based imaging method.
allowed us to show that this interpolation/extrapolation method is clearly advantageous, since it provides reliable reconstructions (particularly in recovering the flux intensity) in a short computational time.

Next developments of this approach to solar X-ray astronomy based on rotating modulation collimators will involve:

- an optimization of the method presented in this paper, whereby, in a preprocessing step, the wild oscillations of the visibility pattern typical of specific events will be reduced by means of a proper shift of the map toward the image centroid (which can be easily estimated directly from the visibilities);
- a systematic application of this approach to a statistical analysis of many visibility sets measured by RHESSI and corresponding to events characterized by different configurations, flux intensities and positions on the solar disk (this kind of analysis will be of course facilitated by the notable computational effectiveness of the method);

We finally observe that the development of efficient techniques for visibility exploitation has certainly a strategical significance, since in the near future new solar missions (GRIPS and STIX, inside Solar Orbiter, overall) have been conceived according to construction paradigms rather similar to RHESSI (STIX will provide a much sparser coverage of the frequency space although the corresponding visibilities will be characterized by a notably better signal-to-noise ratio; on the other hand, GRIPS will do a beautiful work as far as frequency coverage is concerned, although the intensity will be more degraded). Furthermore, accurate visibility-based imaging techniques can be rather easily utilized for data analysis in radioastronomy and image integration at different wavelengths can be utilized to constrain the

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Table 2. Reconstruction errors for the simulated configuration inspired by the July 23 2002 event. The methods applied are our interpolation/extrapolation method (with projected and thresholded iterations) and a maximum entropy method: 'rms' denotes the root mean square error over the entire image; 'pos C', 'pos M', 'pos N' and 'pos S' are the distances between the original and reconstructed left (Coronal), Middle, upper right (North) and bottom right (South) sources; 'peak C', 'peak M', 'peak N' and 'peak S' are the relative errors on the peak intensity of the same sources.
Figure 6. Application of the interpolation/extrapolation algorithm to two real data sets observed by RHESSI. May 21 2004 event: (a) reconstruction when the projected Landweber scheme is applied; (b) reconstruction when the thresholded Landweber scheme is applied. August 23 2005 event: (c) reconstruction when the projected Landweber scheme is applied; (d) reconstruction when the thresholded Landweber scheme is applied.

reconstruction procedure with complementary information coming from different acquisition techniques.

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REFERENCES